We've Got 4.8 stars!

STUDY MATERIAL FOR BTECH EXAMS FOR WORKING PROFESSIONALS

- 100% exam oriented
- Latest questions included
- Frequent updates
- Low fee

MORE INFO

+91-9412903929

AMIESTUDYCIRCLE.COM

AMIESTUDYCIRCLE@GMAIL.COM

CIVIL LINES, NEAR IIT, ROORKEE

BTECH NOTES SERIES

Electrical Machines - I

(DC Machines and Transformers)

(As Per AICTE/Technical Universities Syllabus)

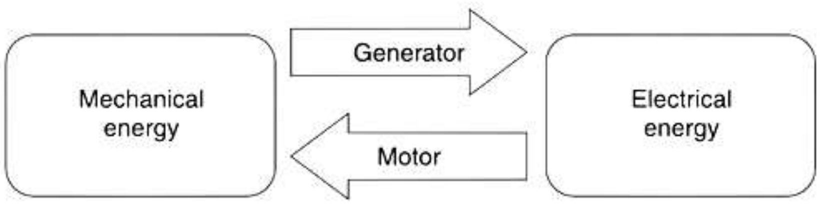
CONTENT

(Use bookmarks to navigate)

PRINCIPLES OF ELECTRO-MECHANICAL ENERGY CONVERSION

PRINCIPLE OF ELECTROMECHANICAL CONVERSION

Electromechanical energy conversion is the transformation of energy between an electrical and a mechanical system. If energy is converted from mechanical to electrical form, the device is called a **generator**, while if electrical energy is converted into mechanical energy, the device is known as a **motor**. Both these effects are shown in following figure.



Electromechanical converters have essentially three features:

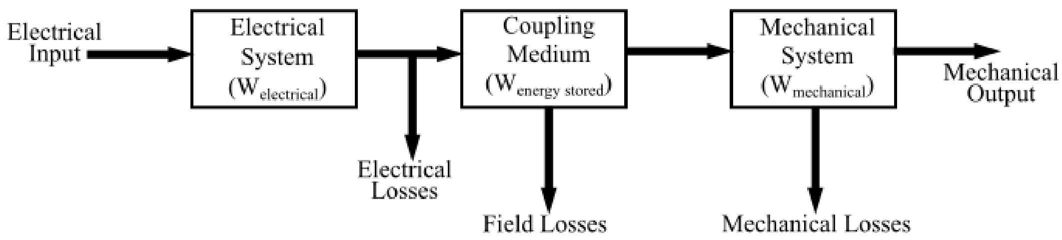
- An electrical system
- A mechanical system
- A coupling field

Electromechanical conversion results from the following two electromagnetic laws given by Faraday:

- When an electric current passes through a conductor, a **magnetic field** is produced.
- When the conductor cuts this magnetic field, an **emf** is induced in that conductor. (Generator action)

Motoring action

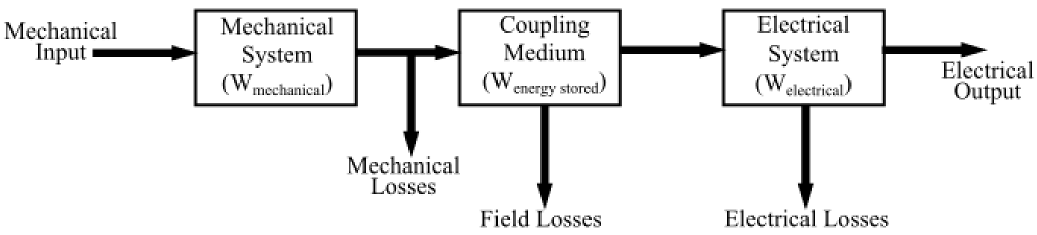
In motoring action, electric current flows through the conductors placed in a magnetic field due to which a force is produced on each conductor. The conductors are placed on a rotor, which is free to move. Therefore, an electromagnetic torque is produced on the rotor so that the rotor starts rotating at some speed.



The torque produced on the rotor is transferred to a shaft of the rotor and hence it can drive a mechanical load. Since the conductors are rotating in a magnetic field, thus an EMF is also induced in each conductor.

Generating action

In generating action, in this case the rotor is driven by a prime mover.



An EMF is induced in the rotor conductors due to which a current will flow and deliver electric power to the load. In addition to this, the current flowing through the conductors will interact with the magnetic field to produce a reaction torque, which will tend to oppose the torque developed by the prime mover.

Energy Balance Equation

The energy balance equation is an expression which shows the complete process of energy conversion. In an electromechanical energy conversion device, the total input energy is equal to the sum of three components :

- Energy dissipated or lost
- Energy stored
- Useful output energy

Therefore, for an **electric motor**, the energy balance equation can be written as,

Electrical energy input = Energy dissipated + Energy stored + Mechanical energy output

Where,

- The electrical energy input is the electricity supplied from the main supply.
- Energy stored is equal to sum of the energy stored in the magnetic field and in the mechanical system in the form of potential and kinetic energies.
- The energy dissipated is equal to sum of energy loss in electric resistance, energy loss in magnetic core (hysteresis loss + eddy current loss) and mechanical losses (windage and friction losses).

For an **electric generator**, the energy balance equation can be written as,

Mechanical energy input = Electrical energy output + Energy stored + Energy dissipated

Where, the mechanical energy input is the mechanical energy obtained from a turbine, engine, etc. to turn the shaft of the generator.

AMPERE'S LAW

Suppose, a coil having N turns is wrapped around the rectangular core of some ferromagnetic material and a current i amperes is passed through it. Due to this current, a magnetic field is produced around that coil. If it is assumed that all the flux produced by the core remains in the core and there is no leakage, its **Ampere’s law** can be given as:

$$Hl = Ni \text{ or } H = Ni/l$$

where H is the magnetic field intensity, and l is the mean length of the core.

The relation between flux density produced in the core and the field intensity is given as:

$$B = \mu H$$

where μ is the **permeability** of the core material.

We see that the actual flux density produced in the material depends on two factors:

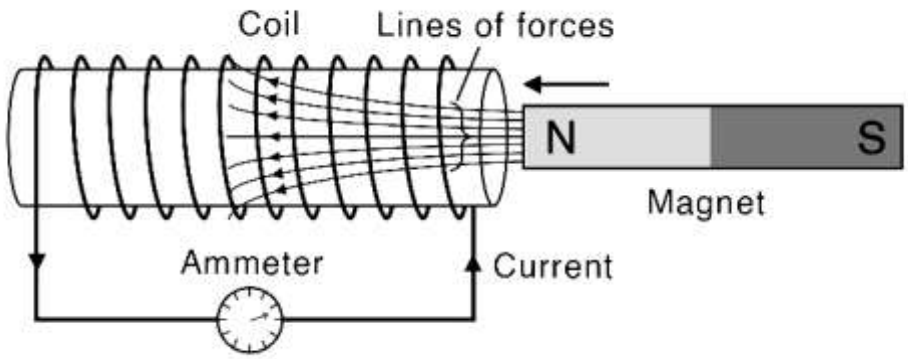
- Magnetic intensity H, which actually is the effort made by the flowing current in the coil to establish the magnetic field,
- Permeability of the core material.

FARADAY’S LAWS OF ELECTROMAGNETIC INDUCTION

First Law

Whenever the flux linking with a coil or closed circuit changes a static emf is induced in it and such an emf lasts only for the time the change is taking place.

Faraday's first law can be well understood from the Figure below. As long as the magnet is moved to and fro, the flux linked with coil changes and hence an emf is induced in the coil, whose presence is indicated by the galvanometer.



As soon as the magnet becomes stationary, the changes in flux is stopped and hence emf in the coil also dies.

It is to be noted that Faraday's first law merely tells us about the condition under which emf is induced in the coil, and not about the magnitude of emf induced. The magnitude of induced emf is given by second law.

Second Law

According to this law, the magnitude of induced emf is equal to the rate of change of flux linked with the closed circuit or coil.

Suppose that a coil with N turns is subjected to a change in flux dϕ in time dt, then the average emf induced between two terminals of the coil is given by

$$e = -N\frac{d\phi}{dt}$$

here emf is in volts, and dϕ/dt is in Wb/s.

The minus sign indicates that the induced emf opposes the changed which produces it, as stated generally in Lenz's law.

LENZ'S LAW

This law gives the direction of induced e.m.f. and hence current. According to this law the direction of induced emf(and hence current) is such that it opposes the cause producing it.

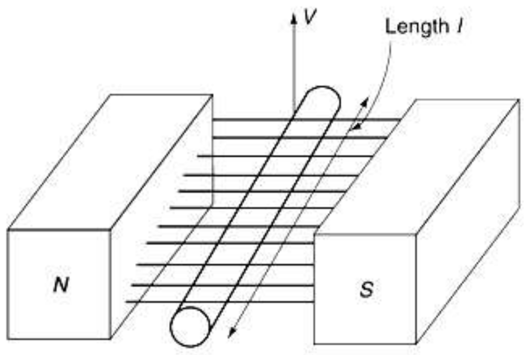
In Figure above, if the N-pole is moved towards the coil, then the current in the coil would be induced in such direction, that the portion of coil facing the N-pole of magnet, would start acting like N-pole and thus it would tend to repel the approaching N-pole of magnet(hence trying to decrease the increasing magnetic flux, i.e. cause).

Again, if the N-pole of magnet is moved away from the coil, then the portion of coil facing the magnet would become S-pole and try to attract the magnet. Hence, the coil tends to increase the decreasing magnetic flux(cause).

DYNAMICALLY INDUCED EMF (GENERATOR ACTION)

When a conductor moves in a magnetic field, some voltage is induced in it, and following expression can be derived.

$$e = Blv$$



This induced emf is called motional emf or dynamically induced emf. All the generators work on this principle.

Now, if a conductor cuts a magnetic field in such a way that it makes an angle θ with the magnetic field, the emf induced in the conductor is given by

$$e = Blv \sin\theta$$

STATICALLY INDUCED EMF (TRANSFORMER ACTION)

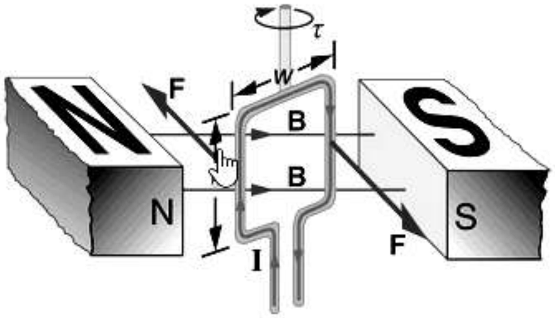
If the conductor does not move but the associated flux varies with time, then the emf induced in the conductor (or coil) is called statically induced emf, which is given by

$$e = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt}$$

A **transformer** works on this principle.

MAGNETIC FORCE (MOTOR ACTION)

This is based on the principle of interaction. Given figure shows a one-turn, carrying-current coil, which is placed in a magnetic field, and the force is induced in that conductor.



The direction of the force can be found by **Fleming's Left Hand Rule or Right Hand Screw Rule**. The force is always in such a direction that the energy stored in the magnetic field is minimum.

The value of the torque is given by

$$F = BIl$$

where B is the flux density, i the current and l is the length of the conductor.

If the conductor moves in the magnetic field in such a way that its axis makes an angle θ with the magnetic field, the force is given by

$$F = BIl \sin\theta$$

Torque $T = wBIl\sin\theta = (A/l)Bil\sin\theta = ABI\sin\theta$

where w is width of current carrying loop and "l" is its length.

If we have a multiple loop of N turns, then

$$T = NABI\sin\theta$$

Then, work done/Revolution = Force x Circumference.

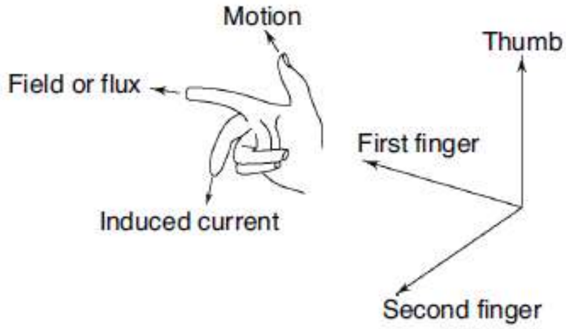
$$= F\text{x } 2\pi R = 2\pi T \text{ (because } T = F \text{ x } R)$$

Mechanical power developed will be given by

$$= 2\pi T x \frac{N}{60} = \frac{2\pi NT}{60} \text{ Watt}$$

FLEMING'S RIGHT-HAND RULE

The direction of the induced emf can be found out by Fleming's right-hand rule as shown in given figure.



This rule states that if anyone spreads the thumb, forefinger and second finger of the right hand so that they are mutually perpendicular to each other, the thumb indicates the direction of rotation of the conductor in the magnetic field, the forefinger shows the direction of the magnetic field and the second finger indicates the direction of the flow of the induced current in the conductor.

MAGNETIC CIRCUIT

Magnetic flux lines form closed loops. A **magnetic circuit** is defined as the closed path followed by flux lines. An **electric circuit** provides a path for electric current, whereas a magnetic circuit provides a path for magnetic flux.

MAGNETOMOTIVE FORCE (MMF)

A current-carrying conductor produces a magnetic field around it. The coil should have the correct number of turns in order to produce the required flux density. Magnetomotive force (mmf) is defined as the product of current and number of turns.

$$\text{Magnetomotive force} = \text{Current} \times \text{Turns}$$

Let I be the current through the coil (A) and N be the number of turns in the coil.

$$F = IN$$

The unit of mmf is ampere-turns (AT). Since N is dimensionless, its unit is amperes (A) also.

MAGNETIC FIELD INTENSITY

Magnetomotive force per unit length of magnetic flux path is called magnetic field intensity (H).

$$H = \frac{F}{l} = \frac{IN}{l} \text{ A/m}$$

where l is the mean length of the magnetic circuit in metres. Magnetic field intensity is also called magnetic field strength or magnetizing force.

From this equation

F = Hl

MAGNETIC FLUX

A current-carrying conductor produces a magnetic field around it. The magnetic field is measured in terms of flux lines (or simply flux) f and flux density B, the number of flux lines per unit area.

$$B = \frac{\phi}{A}$$

The density of flux is greatest near the conductor and tapers off inversely with distance.

$$B = \frac{\mu_0 I}{2\pi r}$$

The unit of flux ϕ is Weber (Wb) and the unit of magnetic intensity (B) is Weber per metre square (Wb/m^2) or tesla (T).

Example

A coil of 100 turns is wound uniformly over a wooden ring having a mean circumference of 500 mm and a uniform cross sectional area of 500 mm^2. If the current through the coil is 2.0 A calculate

(i) the magnetic field strength

(ii) the flux density

(iii) the flux

(iv) mmf

Solution

Mean circumference = 500 mm = 0.5 m

H = NI/l = 100 x 2/0.5 = 400 AT/m or A/m

$B = \mu_0 H = 4\pi \times 10^{-7} \times 400 = 502.65\ \mu T$

$\phi = BA = 502.65 \times 10^{-6} \times 500 \times 10^{-6} = 0.2513\ \mu Wb$

BIOT-SAVART LAW

Biot-Savart law states that at any point P the magnitude of the magnetic field intensity produced by the differential element is proportional to the product of current, magnitude of differential length, and the singe of angle lying between the filament and a line connoting the filament to the point P where the field is desired.

The basic relation for the magnetic flux density, dB at a point P as produced by a current carrying element (See fig.) is

$$dB = \frac{\mu}{4\pi r^2} I.dl \sin\theta$$

μ is permeability of medium. The permeability of vacuum is designated as μ_0 and is equal to 4π x 10^{-7} H/m.

MUTUAL INDUCTANCE

If there are two coils A and B such that the changing flux in one of them, say A, may be linked with other coil, say B, then induced emf would be observed in coil A due to self induction, whereas in coil B, it would be produced due to mutual induction.

Thus, mutually induced emf may be defined as the emf induced in static coil due to changing current(hence flux) in another neighbouring coil.

Mutually induced emf in coil

$B \propto$ rate of change of current in coil A

or $e_M \propto \frac{dI_1}{dt}$, where I, is the current in coil A

or $$e_M = M\frac{dI_1}{dt}$$ **Remember**

Where M is the constant of proportionality called mutual inductance. Its unit is Henry.

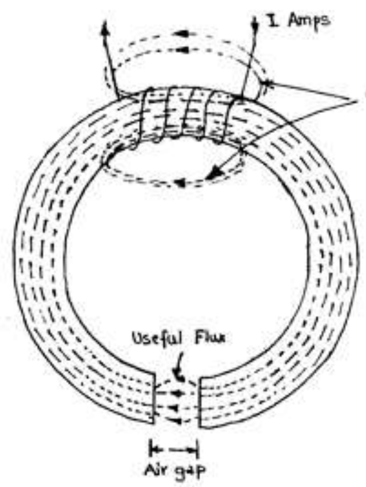
The value of M may either be calculated from the above formula(if induced emf, and value of rate of change of current of first coil is given) or by using the following formula,

$$M = \frac{N_2\phi_1}{I}$$ **Remember**

where, ϕ_1 and I_1 are flux and current in primary coil and N_2 are the turns of secondary coil.

MAGNETIC LEAKAGE AND FRINGING

Magnetic flux which does not follow the intended port in a magnetic circuit is called *leakage flux*. As against this, there is flux(in the air gap) which is utilized for useful purposes. Such flux is called *useful flux*. Both types of fluxes have been shown in Figure.



Hence

Total flux = useful flux + leakage flux

It is assumed for the purpose of calculations that iron carries whole of the flux throughout its entire length.

The ratio of total(flux in the iron path) to the useful flux(flux in air gap) is called *leakage factor*(λ), i.e.

leakage factor(λ) = Flux in iron path(total flux)/ flux in air gap(useful flux)

A reference of above figure will show that useful flux passing across the air gap has a tendency to bulge outwards thus reducing the flux density in the gap. This effect is known as *fringing of flux*.

Example

An electromagnet has an air gap of 6 mm, the flux density in the air gap being 1.257 Tesla. Calculate the number of ampere turns required for the gap.

Solution

Given that $B = 1.257$ T (Wb/m^2), l = 6 mm = 0.006 m

$\mu_0 = 4\pi \times 10^{-7}$ H/m, $\mu_r = 1$(since media is air itself)

We know $B = \mu_0 \mu_r H_{gap}$

$\therefore$ $H_{gap} = 1.257/4\pi \times 10^{-7} = 10^6$ AT/m

Also H_{gap} = MMF/l [Same as E = V/d in electrostatic]

$\therefore$ 10^6 = Ampere turns/0.006

$\therefore$ Ampere turns = 6000

Example

An iron ring of mean circumference 100 cm is made from round iron of cross section 10 cm. Its relative permeability is 300. The ring has a saw cut, 2 mm wide made in it. Find the current required to produce a flux of 0.1 x 10^{-2} Weber, if it is wound with 200 turns.

Solution

Given $\phi = 0.1 \times 10^{-2}$ Wb

Area for iron (a) = 10 cm^2 = 10^{-3} m^2

Now $$B = \frac{\phi}{a} = \frac{0.1x10^{-2}}{10^{-3}} = 1 Wb/m^2$$

$$H_{iron} = \frac{B}{\mu_0\mu_r} = \frac{1}{4\pi x 10^{-7}(300)} AT/m$$

$$\therefore \text{ H for 100 cm} = \frac{100(10^{-2})}{4\pi x10^{-7}(300)} = 2.65x10^3 = 2650 AT$$

Now $$H_{airgap} = \frac{B}{\mu_0\mu_r} = \frac{1}{4\pi x10^{-7}(1)} AT/m$$

$$\text{H for 2 mm gap} = \frac{2x10^{-3}}{4\pi x10^{-7}} = 15.92x100 = 1592 AT$$

Total AT required = 2650 + 1592 = 4242

Number of turns = 200

$\therefore$ Current required = 4242/200 = 21.21 amp

Example (AU 2023, 6 marks)

The magnetic circuit has dimensions, cross sectional area of core = cross sectional area of air-gap = 9 cm^2, air-gap length = 0.050 cm, mean core length = 30 cm and N = 500 turns. Assume the value, μ_r = 70,000 for core material. Find (1) the reluctances of the core and air-gap, for the condition that the magnetic circuit is operating with flux density in the core= 1.0 T, (2) the flux and (3) the current.

Solution

Reluctances

$$R_c = \frac{l_c}{\mu_r\mu_0 A_c} = \frac{0.3}{70,000(4\pi x10^{-7})(9x10^{-4})} = 3.79x10^3\ A.turns/Wb$$

$$R_g = \frac{g}{\mu_0 A_g} = \frac{5x10^{-4}}{(4\pi x10^{-7})(9x10^{-4})} = 4.42x10^5 \ A.turns / Wb$$

Flux

$$\phi = B_c A_c = 1.0(9x10^{-4}) = 9x10^{-4} \ Wb$$

Current

$$i = \frac{F}{N} = \frac{\phi(R_c + R_g)}{N} = \frac{9x10^{-4}(4.46x10^5)}{500} = 0.80 \ A$$

Example (AKTU 2022, 7 marks)

The magnetic flux density on the surface of an iron face is 1.6 T which is a typical saturation level value for ferromagnetic material. Find the force density on the iron face.

Solution

Let the cross-sectional area of the iron face be A.

Consider the field energy in the air gap volume contained between two parallel faces separated by a distance x

$$W_f(B, x) = \frac{1}{2}\frac{B^2 Ax}{\mu_0}$$

The mechanical force due to the field is

$$F_f = -\frac{\partial W_f(B, x)}{\partial x} = -\frac{1}{2}\frac{B^2 A}{\mu_0}$$

The negative sign indicates that the force acts in a direction to reduce x (i.e. it is an attractive force between the two faces).

The force per unit area is

$$|F_f| = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}.\frac{(1.6)^2}{4\pi x10^{-7}} = 1.02x10^6 \ N / m^2$$

ENERGY STORED IN A MAGNETIC FIELD

Suppose a coil of N turns is mounted on a magnetic core and is connected to a voltage source. By applying KVL. the voltage applied is given as

v = iR + e (1)

where e is the voltage induced in the coil or back emf of the coil which opposes the applied voltage and R is the resistance of the coil. The instantaneous power input to the system is given by

$$p = vi = i^2R + ei \qquad (2)$$

Now suppose DC voltage is applied to the circuit at time t = 0 and at end of the T seconds, the current has attained a value of i amperes. The energy input to the system during this interval is

$$W_{EI} = \int_0^T pdt = \int_0^T i^2Rdt + \int_0^T eidt \qquad (3)$$

This equation shows that the input energy consists of **two parts**. The first part is dissipated in the **ohmic field loss** while the second part is the energy stored in the **magnetic field**.

By using Faraday's law,

$$e = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt} \qquad (4)$$

where λ is magnetic flux linkage given by Nϕ.

∴ Field energy is given by

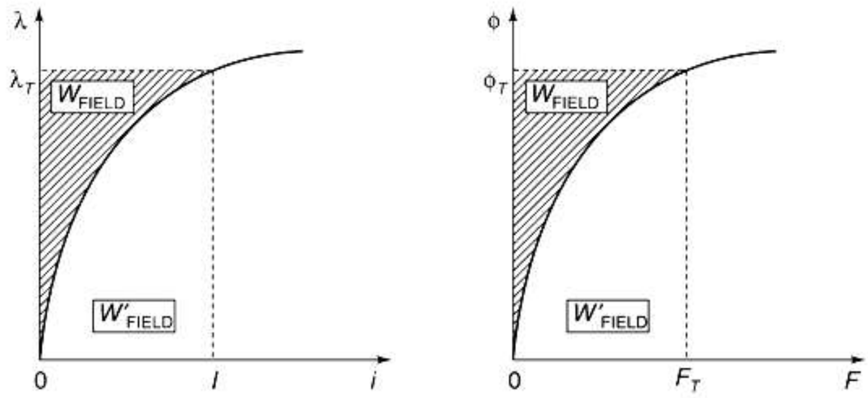
$$W_{Field} = \int_0^T \frac{d\lambda}{dt} idt = \int_0^T id\lambda \qquad (5)$$

Or $$W_{Field} = \int_0^T N\frac{d\phi}{dt} idt = \int_0^T Nid\phi = \int_0^T Fd\phi \qquad (6)$$

where F = Ni is the MMF.

Field Energy

Energy stored in the magnetic field is called **field energy**; it is given by Eqn. 5 and 6 and is shown in Fig. (a) and (b), respectively.



(a) (b)

For a linear system, the field energy is given by

$$W_{Field} = \int_0^{\lambda} i d\lambda = \int_0^{\lambda} \frac{\lambda}{L} d\lambda = \frac{\lambda^2}{2L}$$

Now, λ = Li

$$\therefore \qquad W_{Field} = \frac{(Li)^2}{2L} = \frac{1}{2} Li^2$$

Co-energy

The electrical energy which is converted into magnetic energy is called **co-energy**. From Fig. (a), the area above the magnetization curve (or λ - i) gives the **field energy** while the area below the magnetization curve gives the **co-energy** which is denoted by W'_{Field}. However, *co-energy has no physical significance* but it is used in finite element systems to find mechanical force between magnetized parts.

$$W'_{Field} = \int_0^{i} \lambda di = \int_0^{i} L i di = \frac{1}{2} Li^2$$

Thus, for a linear system, the field energy and co-energy, both are equal.

Thus, for a linear system,

Total energy = Co-energy + Field energy

Example (PTU 2018, 5 marks)

An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux of 1 Wb/m^2, if the relative permeability of iron is 1000. What is the value of energy stored?

Solution

$B = \mu_0\mu_r (NI/l)$

$\therefore \qquad 1 = 4\pi \times 10^{-7} \times 1000 (400I/0.2\pi)$

Giving $\qquad I = 1.25$ A

Now $\qquad L = \mu_0\mu_r A N^2/l = 4\pi \times 10^{-7} \times 1000 \times (100 \times 10^{-4}) \times (400)^2/0.2\pi = 32$ H

Energy $\qquad E = \frac{1}{2} LI^2 = \frac{1}{2} x 3.2 x 1.25^2 = 2.5 J$

EXCITING SYSTEMS

The energy conversion process, the magnetic field plays an important role. We have also seen how the magnetic field is produced in the coil when the current is passed through it. In the electromechanical conversion process, this is called **exciting system**. Mainly two types of excitation systems are used in electrical machines.

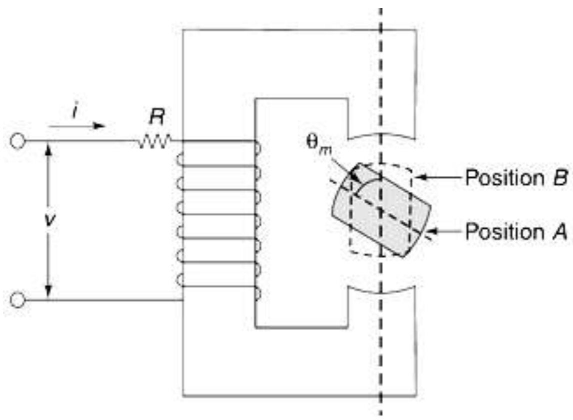
These are:

- Singly excited system
- Doubly excited system.

Before we discuss this system in detail, we will look at how much energy is stored in the magnetic field.

Singly Excited Systems

A simple, singly excited system is shown below.



A coil is wound round a magnetic core and is connected to a voltage source. The rotor will experience a torque where the magnetic field is strong. So, the rotor will try to take a position where the reluctance of the magnetic path is the minimum. This reluctance is dependent on the rotor angle. This torque is called **reluctance torque** or saliency torque due to saliency of the rotor.

Now, suppose R is the resistance of the coil. We apply KVL to get the instantaneous voltage as under

$$v = Ri + \frac{d\lambda}{dt}$$

Multiplying the above equation with current i, we get

$$vi = i^2R + i\frac{d\lambda}{dt}$$

Integrating both sides with respect to time from t = 0 to t = t, assuming that flux linkages and current to be initially zero.

$$\int_0^t vidt = \int_0^t Ri^2 dt + \int_0^\lambda id\lambda$$

We can write the above equation as under:

Total electrical input energy = (Electrical energy losses) + (Useful electrical energy)

or $W_{EI} = W_{ELOSS} + (W_{EFIELD} + \mu_{MECH})$

and $\int_0^\lambda id\lambda = W_{EFIELD} + W_{MECH} = W_{FIELD}$

Electromagnetic torque is given by,

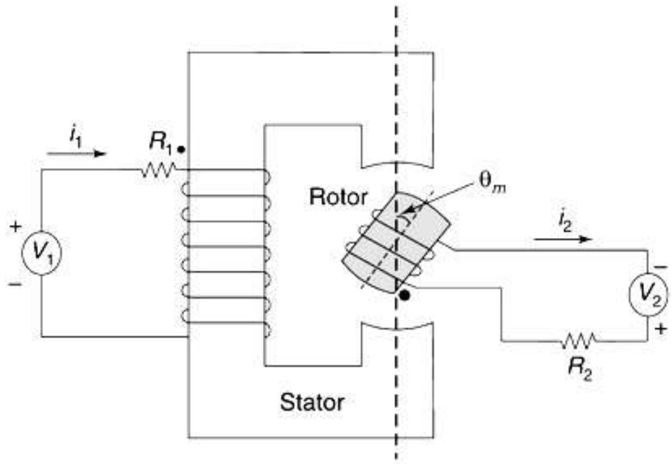
$$T_e = \lim_{\Delta\theta_m \to 0} \left\{ -\frac{\Delta W_{EFIELD}}{\Delta\theta_m} \right\}_{\lambda, const} = \left\{ -\frac{\partial}{\partial\theta_m} \frac{\lambda^2}{2L} \right\}_{\lambda, const}$$

$\therefore$ $$T_e = \frac{\lambda^2}{2} \cdot \frac{1}{L^2} \cdot \frac{\partial L}{\partial\theta_m}$$

$\therefore$ $$T_e = \frac{i^2}{2} \frac{\partial L}{\partial\theta_m}$$

Doubly Excited Systems

A doubly excited system is shown in following figure.



In this case, there are two independent sources of excitation such as DC separately excited generator, synchronous motor, loud speakers, tachometers, etc. Consider that both the stator and the rotor have saliency.

The flux linkage equations for the two winding are

$$\lambda_1 = L_1 i_1 + M i_2 \quad (1)$$

and $$\lambda_2 = L_2 i_2 + M i_1 \quad (2)$$

The instantaneous voltage equations for the two coils are

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \quad (3)$$

and $$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} \quad (4)$$

$\therefore$ $$v_1 = R_1 i_1 + \frac{d}{dt}(L_1 i_1) + \frac{d}{dt}(M i_2) \quad (5)$$

and $$v_2 = R_2 i_2 + \frac{d}{dt}(L_2 i_2) + \frac{d}{dt}(M i_1) \quad (6)$$

From these equations, we finally get following **power equations for the coils**

$$v_1 i_1 = R_1 i_1^2 + L_1 i_1 \frac{di_1}{dt} + i_1^2 \frac{dL_1}{dt} + i_1 M \frac{di_2}{dt} + i_1 i_2 \frac{dM}{dt} \quad (7)$$

and $$v_2 i_2 = R_2 i_2^2 + L_2 i_2 \frac{di_1}{dt} + i_2^2 \frac{dL_1}{dt} + i_2 M \frac{di_2}{dt} + i_1 i_2 \frac{dM}{dt} \quad (8)$$

Integrating these equations with respect to time and adding, we finally get the energy equation for the doubly-excited magnetic system. It shows that the total electrical energy input to the system is equal to the sum of two parts, where the first part is the energy to electrical losses and the second is useful electrical energy, i.e.

$$W_e = W_{ELOSS} + (W_f + W_m)$$

where W_e is total electrical energy input and $(W_f + W_m)$ is useful electrical energy.

$$= \int (L_1 i_1 d_{i1} + L_2 i_2 d_{i2} + i_1 M di_2 + 2 i_1 i_2 dM + i_1^2 dL_1 + i_2^2 dL_2 + i_2 M di_1) \quad (9)$$

Hence, the terms dL_1, dL_2 and dM become zero.

Hence, **Energy Stored in the Magnetic Field**

$\therefore$ $$\int dW_{EFIELD} = \int_0^{i_1} L_1 i_1 d_{i1} + \int_0^{i_2} l_2 i_2 d_{i2} + \int_0^{i_1, i_2} (i_1 M di_2 + i_2 M di_1)$$

$$= \frac{1}{2} l_1 i_1^2 + \frac{1}{2} l_2 i_2^2 + M i_1 i_2 \quad (10)$$

Electromagnetic torque

When the rotor rotates, the rate of change of field energy with respect to time is given by differentiating the eqn.(10),

$$\frac{dW_{EFIELD}}{dt} = \frac{1}{2}L_1\frac{di_1^2}{dt} + \frac{1}{2}i_1^2\frac{dL_1}{dt} + \frac{1}{2}L_2\frac{di_2^2}{dt} + \frac{1}{2}i_2^2\frac{dL_2}{dt} + i_1i_2\frac{dM}{dt} + Mi_1\frac{di_2}{dt} + Mi_2\frac{di_1}{dt} \quad (11)$$

We get after integrating

$$W_f = \int L_1i_1d_1 = \frac{1}{2}i_1^2dl_1\int l_2i_2d_2 + \frac{1}{2}i_2^2dl_2 + i_1i_2dM + Mi_1di_2 + Mi_2di_1 \quad (12)$$

The eqn. (11) is a general equation for a moving transducer in which L_1, L_2, M, i_1 and i_2 are all varying with position and time. Now, comparing eqn.(11) with eqn.(9), we obtain,

$$W_m = \int\left(\frac{1}{2}i_1^2dL_1 + \frac{1}{2}i_2^2dL_2 + i_1i_2dM\right) \quad (13)$$

Differentiating eqn.(13) with respect to rotor angle θ_m, we get

$$\frac{dW_m}{d\theta_m} = \int\left(\frac{1}{2}i_1^2\frac{dL_1}{d\theta_m} + \frac{1}{2}i_2^2\frac{dL_2}{d\theta_m} + i_1i_2\frac{dM}{d\theta_m}\right)$$

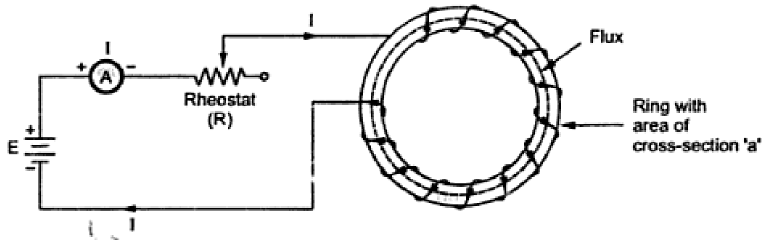
$$\therefore \quad Torque = \frac{1}{2}i_1^2\frac{dL_1}{d\theta_m} + \frac{1}{2}i_2^2\frac{dL_2}{d\theta_m} + i_1i_2\frac{dM}{d\theta_m} \quad (14)$$

The first two terms of the equation (14) are the **reluctance torque** and the last term is known as the **co-alignment torque**, that is due to the two superimposed magnetic fields try to align.

B-H CURVE FOR MAGNETIC MATERIAL

We have already seen that magnetic field strength H is NI/l. As current in coil changes, magnetic field strength also changes. Due to this, flux produced and hence the flux density also changes. So there exists a particular relationship between B and H for a material which can be shown on the graph.

Let us obtain the B-H curve experimentally for a magnetic material. The arrangement required is shown in the Fig.

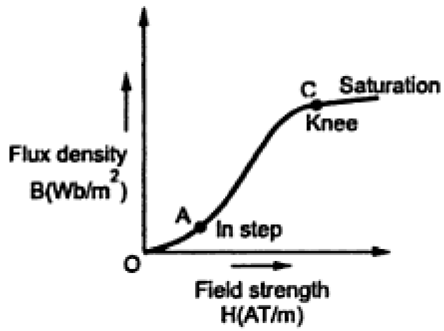


The ring specimen as a mean length of T metres with a cross-sectional area of 'a' square metres. Coil is wound for 'N' turns carrying a current T which can be varied by changing the variable resistance 'R' connected in series. Ammeter is connected to measure the current. For measurement of flux produced, fluxmeter can be used which is not shown in the figure above.

So H can be calculated as NI/l while B can be calculated as ϕ/a for various values of current and plotted.

With the help of resistance R, I can be changed from zero to maximum possible value.

The B-H curve takes the following form, as shown in the following figure.



The graph can be analysed as below :

- **Initial portion** : Near the origin for low values of 'H', the flux density does not increase rapidly, This is represented by curve OA. The point A is called as instep.
- **Middle portion** : In this portion as *H' increases, the flux density B increases rapidly. This is almost straight line curve. At point 'C it starts bending again. The point 'C where this portion bends is called as knee point.
- **Saturation portion** : After the knee point, rate of increase in 'B' reduces drastically. Finally the curve becomes parallel to 'X' axis indicating that any increase in 'H' hereafter is not going to cause any change in 'B\ The ring is said to be saturated and region as saturation region.

We have seen already that according to molecular theory of magnetism, when all molecular magnets align themselves in the same direction due to application of H, saturation occurs. Such curves are also called **saturation curves**.

The materials in which molecular magnets align themselves due to application of H are called ferromagnetic materials. Such materials are iron, cobalt, nickel etc. The above discussed saturated type of magnetization characteristics is the feature of such ferromagnetic materials.

Practical Use of B-H Curves

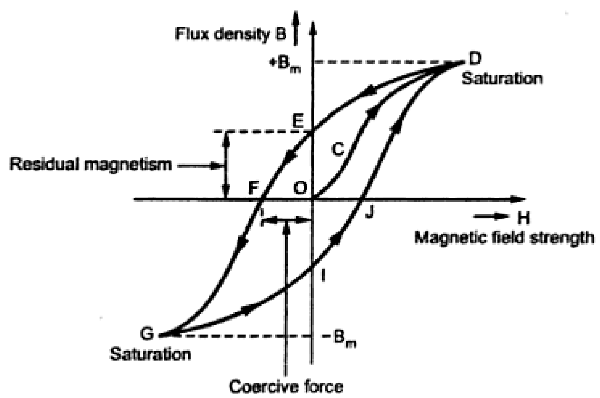
While designing the magnetic circuits, magnetization curves are useful to design the values of B corresponding to H. From this, proper material with required relative permeability can be selected.

The various materials like iron, steel are generally represented by the B-H curves and Hr-H curves.

Magnetic Hysteresis/Hysteresis Loop

Magnetic hysteresis is extension to the same concept of B-H curve. Instead of plotting B-H curve only for increase in current if plotted for one complete cycle of magnetization (increase in current) and demagnetization (decrease in current) then it is called hysteresis curve or hysteresis loop.

Following is Hysteresis loop.



Part of Curve **Represents What?**

O-C-D : Region corresponding to normal magnetization curve increased form 'O' to I_{max} corresponding to B_{max} Maximum flux density is $+B_m$.

D-E : Current reduced to zero, but core cannot be completely demagnetized. O-E represents residual magnetism and residual flux density, denoted by $+ B_r$.

E-F : Current is reversed and increased in reversed direction to get complete demagnetization of the core. O-F represent coercive force required to completely wipe out $+ B_r$

F-G Current is increased in reversed direction till saturation in opposite direction is achieved. Maximum flux density same but with opposite direction i.e. $– B_m$.

G-I Current is reduced to zero but again flux density lags and core cannot be completely demagnetized. O-I represents residual flux density in other direction i.e. $– B_r$

I-J Current is again reversed and increased till complete demagnetization is achieved.

J-D Current is again increased in original direction, till saturation is reached. Corresponding flux density is again + B_m.

As seen from the loop 'O-C-D-E-F-G-I-J-D' shown in the figure, the flux density B always lags behind the values of magnetic field strength H. When H is zero, corresponding flux density is + B_r This effect is known as **hysteresis**.

Hysteresis Loss

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization it results into disturbance in (he alignment of the various domain. Now energy gets stored when magnetic field is established and energy is returned when field collapses. But due to hysteresis, all the energy is never returned though field completely collapses. This loss of energy appears as heat in the magnetic material. This is called as hysteresis loss. So disturbance in the alignment of the various domains causes hysteresis loss to take place. This hysteresis loss is undesirable and may cause undesirable high temperature rise due to heat produced. Due to such loss overall efficiency also reduces.

Such hysteresis loss depends on the following factors

- The hysteresis loss is directly proportional to the area under the hysteresis curve i.e. area of the hysteresis loop.
- It is directly proportional to frequency i.e. number of cycles of magnetization per second.
- It is directly proportional to volume of the material. It can be shown that quantitatively the hysteresis loss in joules per unit volume of the material in one cycle is equal to the area of the hysteresis loop.

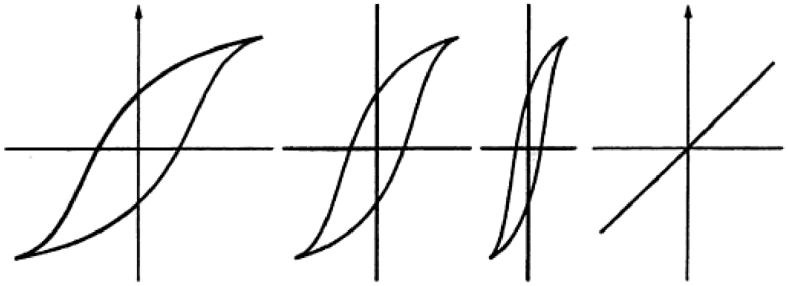
Practical Use of Hysteresis Loop

As we have seen that hysteresis loss is undesirable as it produces heat which increases temperature and also reduces the efficiency.

In machines where the frequency of the magnetization and demagnetization cycle is more, such hysteresis loss is bound to be more.

So selection of the magnetic material in such machines based on the hysteresis loss. Less the hysteresis loop area for the material, less is the hysteresis loss.

Shapes of hysteresis loops for different materials are shown in following figure.



The Fig. (a) shows loop of hard steel, which is magnetic material.

The Fig. (b) shows loop of cast steel.

The Fig. (c) shows loop of permalloy (Alloy of nickel and iron) i.e. ferromagnetic materials.

The Fig. (d) shows loop for air or non magnetic material.

The materials iron, nickel, cobalt and some of their alloys and compounds show a strong tendency to move from weaker to stronger portion of a non-uniform magnetic field. Such substances are called **ferromagnetic** materials.

The hysteresis loss is proportional to the area of the hysteresis loop. For ferromagnetic materials the hysteresis loop area is less as shown in the Fig. (c) thus hysteresis loss is less in such materials.

In nonmagnetic materials, the hysteresis loop is straight line having zero area hence hysteresis loss is also zero in such materials.

COMPARISON BETWEEN ELECTRIC & MAGNETIC CIRCUIT

Electric Circuit	Magnetic Circuit
Current(I) = E.M.F.(V)/Resistance(R)	Flux(θ) = MMF/Reluctance(S)
Resistance(R) = $\rho l/A = 1/\sigma a$ where σ is conductivity.	Reluctance(S) = $l/\mu A = l/\mu_0\mu_r a$

LINEAR VS. NONLINEAR CIRCUITS

- **Linear circuits** follow the principles of linearity, which means that the output of the circuit is directly proportional to its input.
- **Nonlinear circuits**, on the other hand, do not follow the principles of linearity. In these circuits, the output is not directly proportional to the input, and the properties of homogeneity and superposition do not hold.

Examples

- The examples of linear circuits are resistance and resistive circuit, inductor and inductive circuit and capacitor and capacitive circuit.
- Some of the examples of nonlinear circuit of nonlinear elements are diode, transformer, iron core, inductor, transistor etc.

Differences

- **Linearity**: Linear circuits follow the principles of linearity, while nonlinear circuits do not.
- **Homogeneity** and Superposition: Linear circuits obey the properties of homogeneity and superposition, whereas nonlinear circuits do not.
- **Behaviour**: Linear circuits exhibit predictable behaviour, while nonlinear circuits can exhibit more complex, unpredictable behaviour.
- **Components**: Linear circuits primarily consist of resistors, capacitors, and inductors, while nonlinear circuits often include diodes, transistors, and other nonlinear components.

ASSIGNMENT

Q.1. (AU 2022, GTU 2021, 2 marks): Compare magnetic and electric circuits.

Answer: Described in this module.

Q.2. (AKTU 2018, 2 marks): Explain Faraday's laws of electromagnetic induction and Lenz's law.

Answer: Described in this module.

Q.3. (AKUB 2020, RTU 2018, 2023, 2 marks): State Biot-Savart law. Write its applications.

Answer: Described in this module.

Q.4. (AKUB 2020, RTU 2023, 2 marks): State Ampere law.

Answer: Described in this module.

Q.5. (PTU 2018, RTU 2023, 2 marks): Explain Fleming’s right hand rule.

Answer: Described in this module.

Q.6. (AU 2022, 2 marks): Derive the expression for the force and torque on a Current Carrying Conductor.

Q.7. (RTU 2018, 2 marks): What is mmf and flux?

Answer: Described in this module.

Q.8. (RTU 2023, 2 marks): What is an air gap flux?

Answer: Described in this module.

Q.9. (RTU 2023, 2 marks): Give name of sources that store energy in magnetic circuits.

Answer: Inductor stores energy in the form of the magnetic field. Capacitor stores energy in the form of the electric field. So that transients are present in the inductor & capacitor.

Q.10. (RTU 2023, 2 marks): Give example of linear and non-linear magnetic circuits.

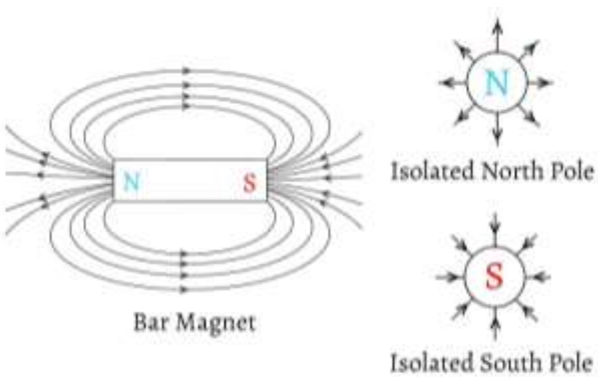
Answer: Described in this module.

Q.11. (RTU 2018, 15 marks): Compare linear and non-linear magnetic circuit and also discuss the flux linkage versus current characteristics of magnetic circuits.

Answer: Described in this module.

Q.12. (AKUB 2020, 2 marks): Draw magnetic lines of a magnate showing magnetic flux lines clearly.

Answer: Magnetic Field lines are imaginary lines along which North Magnetic Pole would move. In a bar magnet, the magnetic field lines look like



Q.13. (RTU 2018, 8 marks): Enumerate the influence of highly permeable material on the magnetic flux lines.

Answer:

- Concentration: This concentration amplifies the magnetic field intensity within the material.
- Guiding: Magnetic flux lines tend to follow the path of least resistance. Since the highly permeable material offers an easier path for the field lines compared to surrounding space, they preferentially travel through it.
- Shielding: As the lines are drawn into the highly permeable material, they are less present in the surrounding space.
- Circuitry: In applications like transformers and electromagnets, highly permeable materials are used to channel and direct the magnetic field efficiently.

Q.14. (PTU 2020, 2 marks): Which material is suitable for making permanent magnet? Give reason.

Answer: Usually ferromagnetic or ferrimagnetic materials are used for making permanent magnets these materials includes iron, nickel, cobalt and some rare earth metals. These materials were exposed to strong magnetic fields until it retains its magnetic field.

Q.15. (RTU 2018, 2 marks): What is the importance of electromagnet in machine?

Answer: Electromagnetism has important scientific and technological applications. It is used in many electrical appliances to generate desired magnetic fields. It is even used in a electric generator to produce magnetic fields for electromagnetic induction to occur. It has many more technological applications including MRI scanning (magnetic resonance imaging) and electric bells.

Uses of Electromagnets: Generators, motors, and transformers, Electric buzzers and bells, Headphones and loudspeakers, Relays and valves, Data storage devices like VCRs, tape recorders, hard discs, etc., Induction cooker, Magnetic locks, MRI machines, Particle accelerators, Mass spectrometers.

Q.16. (PTU 2020, 2 marks): Why laminated core in electrical machines are used.

Answer: Eddy current losses can be reduced by splitting the solid core into thin sheets called laminations, in the plane parallel to the magnetic field. Each lamination is insulated from each other by a thin layer of coating of varnish or oxide film. By laminating the core, the area of each section is reduced and hence the induced emf also reduces. As the area through which the current is passed is smaller, the resistance of eddy current path increases.

Q.17. (PTU 2020, 2 marks): While comparing magnetic and electric circuit, the flux of magnetic circuit is compared with which parameter of the electric circuit.

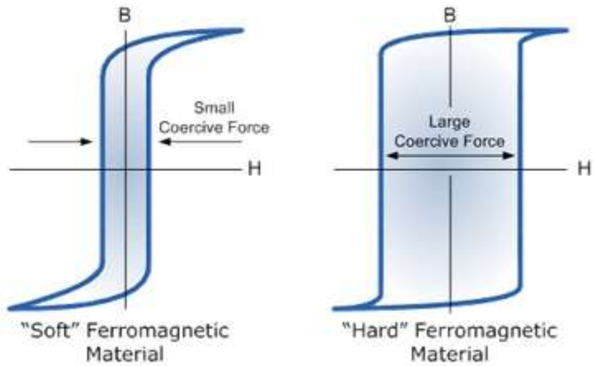
Answer: Current

Q.18. (GTU 2021, 4 marks): What is the eddy current and hysteresis loss? How they can be minimized?

Answer: Described in this module.

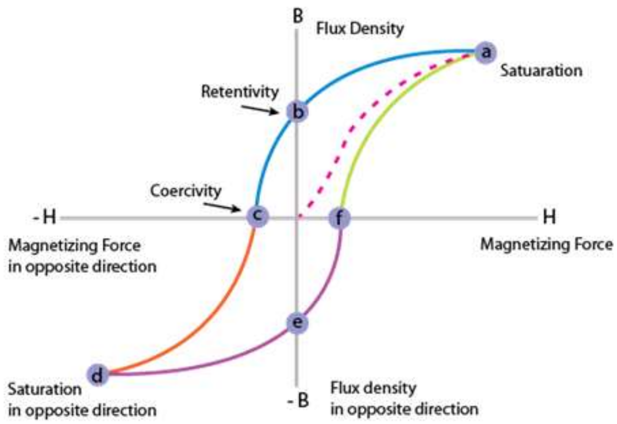
Q.19. (AKUB 2020, RTU 2018, 2023, 2 marks): On the same platform, draw the B-H curve for soft as well as hard magnetic material.

Answer:



Q.20. (GTU 2020, 3 marks): Explain the magnetization curve of ferromagnetic material.

Answer:



Q.21. (RTU 2018, GTU 2020, 3 marks): Derive an expression for energy stored in the magnetic field.

Answer: Described in this module.

Q.22. (AU 2022, 2 marks): Describe multiple excited magnetic field system.

Answer: Described in this module. Describe doubly excited magnetic field system.

Q.23. (AU 2023, 2 marks): Differentiate statically induced EMF from dynamically induced EMF. Give one example for each.

Answer:

- **Statically Induced EMF** is produced when there is no relative movement between conductor and the Source of the magnetic field. Rather, magnetic field itself varies in magnitude. this rate of change of magnetic field around the conductor causes the EMF induction in the conductor. Example of such EMF is the self and mutually induced EMFs in inductors when supplied with Alternating Current.
- **Dynamically Induced EMF** is produced when there exists relative movement between conductor and the Source of the Magnetic field. Here Magnetic field lines does not change in magnitude rather it moves with respect to the conductor, and when it cuts the conductor an EMF is induced in the conductor. Example of such EMF is the EMF induced in the armature of a DC Generator.

Q.24. (AKUB 2020, 2 marks): With the help of neat and labelled diagram, show energy and co-energy.

Answer: Described in this module.

Q.25. (AKTU 2019, 2 marks): Write the energy balance equation for the generator and motor.

Answer: Described in this module.

Q.26. (AKTU 2022, AKUB 2020, MAKUT 2019, 6 marks): Draw and discus a general representation of an electromechanical energy conversion device. Further, explain lossless electromechanical energy conversion.

Answer: Described in this module.

Q.27. (AKTU 2022, 2023, 2 marks): Why do all practical energy conversion devices make use of magnetic field as a coupling medium rather than an electric field? Also, Write the energy balance equation.

Answer: the magnetic field is used as the coupling medium between electrical and mechanical medium because the energy storing capacity of the magnetic field is much higher than the electric field.

Q.28. (AU 2022, AKTU 2023, GTU 2020, 2 marks): Develop a block diagram indicating the process of electromechanical energy conversion.

Answer: Described in this module.

Q.29. (AKTU 2023, GTU 2018, 7 marks): Explain the flow of energy in electromechanical devices for motoring action & generating action with necessary block diagram.

Answer: described in this module.

Q.30. (AKTU 2019, 7 marks): Define energy and co-energy. What is the significance of co-energy? Show that the field energy in a linear magnetic system is given by $W_f = \frac{1}{2} l_i^2 = \frac{1}{2}\psi_i = \frac{1}{2L}\psi^2$

Answer: Described in this module.

Q.31. (AKTU 2022, 7 marks): Show that the magnetic force f_e is given by the expression

$$f_e = -\frac{\partial W_{fld}}{\partial x}(\psi, x) = -\frac{\partial W_{fld}}{\partial x}(\phi, x)$$

Answer: Described in this module.

Q.32. (AKTU 2022, 10 marks): For an electromagnetic system, show that the mechanical work done is equal to the area enclosed between the two magnetization curves at open and closed positions of the armature and the ψ-i locus during instantaneous armature movement.

Answer: Described in this module.

Q.33. (GTU 2020, 4 marks): Explain singly excited magnetic system.

Answer: Described in this module.

Q.34. (AKTU 2023, AU 2023, GTU 2020, 10 marks): Distinguish between singly-excited and doubly-excited systems. For a singly excited linear magnetic system, derive an expression for the electromagnetic torque.

Answer: Described in this module.

Q.35. (AKTU 2022, AU 2023, GTU 2021, 10 marks): Derive a relationship between magnetic field energy and co-energy for a singly excited system.

Answer: Described in this module.

Q.36. (AKTU 2019, AU 2023, 7 marks): Derive an expression for the torque in a doubly excited system having salient pole type of stator as well as rotor.

Answer: Described in this module.

Q.37. (AKTU 2023, 10 marks): Discuss the components of electro-mechanical torque produced in a doubly excited system with suitable derivation.

Answer: Described in this module.

Q.38. (AKTU 2022, 7 marks): The magnetic flux density on the surface of an iron face is 1.6 T which is a typical saturation level value for ferromagnetic material. Find the force density on the iron face.

Answer: Solved in this module.

Q.39. (AU 2023, 6 marks): The magnetic circuit has dimensions, cross sectional area of core = cross sectional area of air-gap = 9 cm^2, air-gap length = 0.050 cm, mean core length = 30 cm and N = 500 turns. Assume the value, μ_r = 70,000 for core material. Find (1) the reluctances of the core and air-gap, for the condition that the magnetic circuit is operating with flux density in the core= 1.0 T, (2) the flux and (3) the current.

Answer: Solved in this module.

Q.40. (PTU 2018, 5 marks): An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux of 1 Wb/m^2, if the relative permeability of iron is 1000. What is the value of energy stored?

Answer: Solved in this module.

Q.41. (PTU 2020, 5 marks): A coil of 100 turns is wound uniformly over a wooden ring. The ring is having a mean circumference of 500 mm and a uniform cross section area of 400 mm^2. A current of 4 A passed through the coil. Calculate:

(i) magnetic field strength

(ii) flux density

(iii) total flux

Answer: Similar problem is solved in this module.

Q.42. (AU 2022, 6 marks): Calculate the current requires to produce a flux of 1.75 m. Wb in the ring if the relative permeability of the iron is 900, number of turns N = 600 and radius of the cross section r = 3.5 cm.

Answer: 0.15 A

Hint: $B = \frac{\mu_0 \mu_r NIA}{l} Wb / m^2$